Solutions to problem sets 1 & 2

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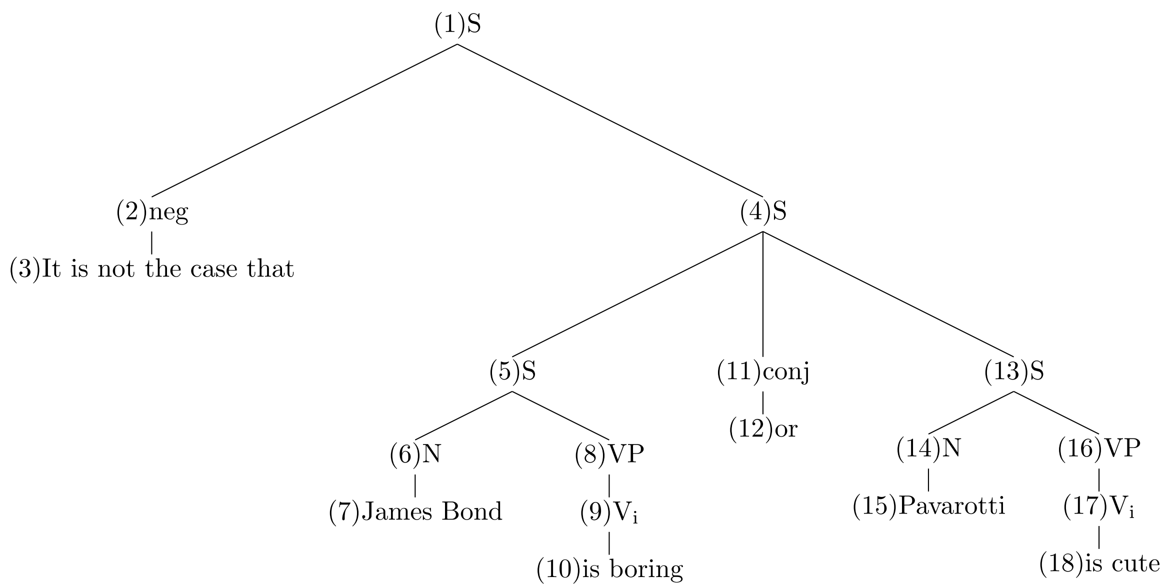
September 22, 2014

The following is a derivation of the truth conditions of (one tree of) 'It is not the case that James Bond is boring or Pavarotti is cute,' using the bottom-up method.



1.	$[\![(18)]\!]^v=\{x{:}x \text{ is cute in } v\}$	lexicon
2.	$[\![(16)]\!]^v=\{x{:}x \text{ is cute in } v\}$	1,pass-up
3.	$[\![(15)]\!]^v=\text{Pavarotti}$	lexicon
4.	$[\![(14)]\!]^v=\text{Pavarotti}$	5,pass-up
5.	$[\![(13)]\!]^v=1$ iff Pavarotti $\in \{x{:}x \text{ is cute in } v\}$	31(a),2,4

By exactly parallel reasoning we can derive:

. . .

10.	$[\![(5)]\!]^v=1$ iff James Bond $\in \{x{:}x \text{ is boring in } v\}$	31(a),7,9

Next we want to derive $[\![(4)]\!]^v$. A first step is to apply our rule for sentences of type [$_S$ S conj S]:

11. $[\![(4)]\!]^v=1$ iff $[\![(11)]\!]^v(< [\![(5)]\!]^v\ ,\ [\![(13)]\!]^v >)=1$

A natural next step is to apply the lexical entry for ‘or’, plus pass-up:

12. $[\![(4)]\!]^v=1$ iff $[\![(5)]\!]^v=1$ or $[\![(13)]\!]^v=1$ lexicon,pass-up

Now we should substitute back in the results of our previous derivations of the semantic values of nodes (5) and (13):

13. $[\![(4)]\!]^v=1$ iff James Bond $\in \{x{:}x$ is boring in $v\}$ or Pavarotti $\in \{x{:}x$ is cute in $v\}$ 5,10,12

This is a fully explicit statement of the semantic value of node (4): notice that the statement of the conditions under which (4) is true does not mention the semantic values of any other nodes. Now all we need to do is move from this to get the semantic value of the root node. As above, the best method is to first state the rule by which the semantic value of node (1) is determined, and then substitute in the results we have already derived. Our rules tell us:

14. $[\![(1)]\!]^v=1$ iff $[\![(2)]\!]^v([\![(4)]\!]^v)=1$

We then apply the lexical entry for ‘It is not the case that’, plus pass-up, to give us

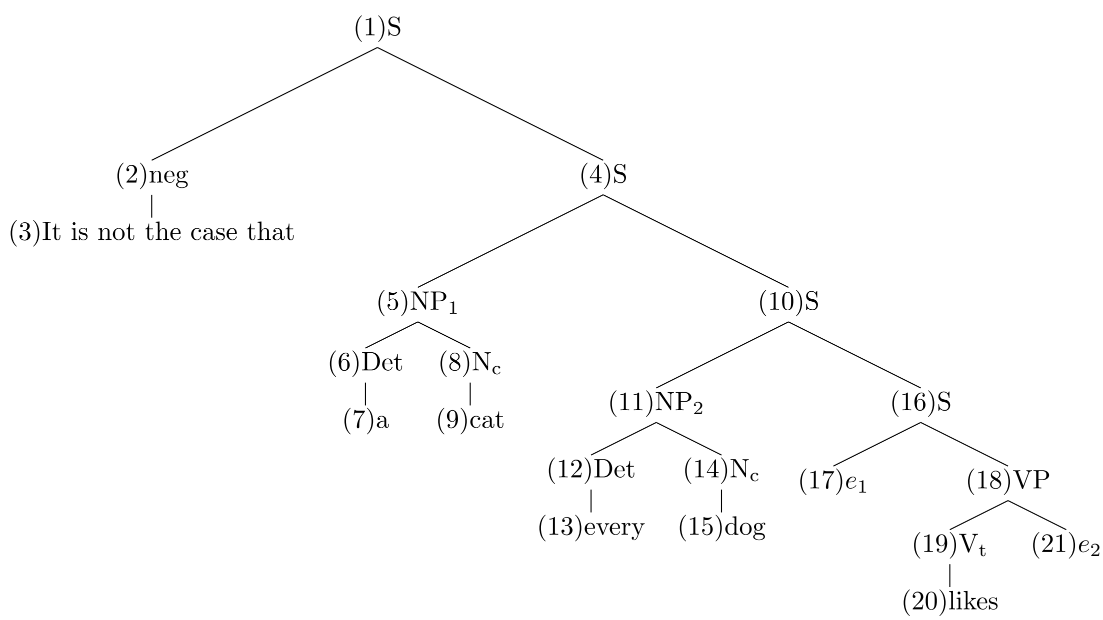
15. $[\![(1)]\!]^v=1$ iff $[\![(4)]\!]^v=0$ lexicon,pass-up

We now substitute back in our derivation of the semantic value of node (4) to give us

16. $[\![(1)]\!]^v=1$ iff James Bond $\notin \{x{:}x$ is boring in $v\}$ and Pavarotti $\notin \{x{:}x$ is cute in $v\}$ 13,15

And we are done.

Now let’s look at the example from problem set 2, ‘It is not the case that a cat likes every dog.’ One tree for this sentence is:



This derivation is much easier if we use the top-down method, and that is what I will use below. Remember that the key here is to begin with the root node, and progressively use the rules and the lexicon to arrive at a fully explicit statement of the sentence's truth conditions. We start out at the top, by applying the rule for negation:

1. $[\![(1)]\!]^{M,g}$=1 iff $[\![(2)]\!]^{M,g}([\![(4)]\!]^{M,g}$=1
2. $[\![(1)]\!]^{M,g}$=1 iff $[\![(4)]\!]^{M,g}$=0

It is now time to make explicit the semantic contribution of the quantifier phrase, 'a cat.' The lexical entry for 'a' tells us what it takes for a quantifier phrase like this to combine with a sentence to form a truth; but the entry also tells us what it takes for the sentence formed to have the value 0. We can use that fact to get to the following line:

3. $[\![(1)]\!]^{M,g}$=1 iff it is not the case that for some $u \in U$, $u \in [\![\text{cat}]\!]^{M,g}$ and $[\![10]\!]^{M,g^{[u/e_1]}}$=1

Now we need to make explicit the semantic contribution of 'every dog.' That is done in just the same way as the previous step; the only additional complication is that we are modifying an already-modified assignment function, to give us a doubly-modified assignment function:

4. $[\![(1)]\!]^{M,g}$=1 iff it is not the case that for some $u \in U$, $u \in [\![\text{cat}]\!]^{M,g}$ and for every $u* \in U$, if $u* \in [\![\text{dog}]\!]^{M,g}$, then $[\![16]\!]^{M,g^{[u/e_1[u*/e_2]}}$=1

The next step is to figure out when $[\![16]\!]^{M,g^{[u/e_1[u*/e_2]}}$=1. This looks daunting, but really this is just a simple sentence of the sort we have known how to handle for a while – a sentence of the form [$_S$ N VP]. It's just that instead of names we now have traces whose semantic values are fixed by the relevant assignment function. But this really does not make things much harder. Using our rule for sentences of this form gives us:

5. $[\![(1)]\!]^{M,g}$=1 iff it is not the case that for some $u \in U$, $u \in [\![\text{cat}]\!]^{M,g}$ and for every $u* \in U$, if $u* \in [\![\text{dog}]\!]^{M,g}$, then $[\![e_1]\!]^{M,g^{[u/e_1[u*/e_2]}} \in [\![(18)]\!]^{M,g^{[u/e_1[u*/e_2]}}$

We now apply our rule for transitive verbs:

6. $[\![(1)]\!]^{M,g}$=1 iff it is not the case that for some $u \in U$, $u \in [\![\text{cat}]\!]^{M,g}$ and for every $u* \in U$, if $u* \in [\![\text{dog}]\!]^{M,g}$, then $[\![e_1]\!]^{M,g^{[u/e_1[u*/e_2]]}} \in \{x :< x, [\![e_2]\!]^{M,g^{[u/e_1[u*/e_2]]}} > \in [\![\text{likes}]\!]^{M,g^{[u/e_1[u*/e_2]}}\}$

Now all that is left to do is figure out the semantic values of the leaves. First, the traces. This is easy, since the semantic value of a trace under an assignment is just the value that that assignment gives to that trace. This gives us:

7. $[\![(1)]\!]^{M,g}$=1 iff it is not the case that for some $u \in U$, $u \in [\![\text{cat}]\!]^{M,g}$ and for every $u* \in U$, if $u* \in [\![\text{dog}]\!]^{M,g}$, then $u \in \{x :< x, u* > \in [\![\text{likes}]\!]^{M,g^{[u/e_1[u*/e_2]}}\}$

Now we just plug in what the lexicon tells us about the non-trace leaves, and we are done:

8. $[\![(1)]\!]^{M,g}$=1 iff it is not the case that for some $u \in U$, $u \in \{x : x$ is a cat in $v\}$ and for every $u* \in U$, if $u* \in \{x : x$ is a dog in $v\}$, then $u \in \{x :< x, u* > \in \{< y, z >: y$ likes z in $v\}\}$